EINSTEIN'S THEORY OF RELATIVITY CONSIDERED FROM THE EPISTEMOLOGICAL STANDPOINT

I

CONCEPTS OF MEASURE AND CONCEPTS OF THINGS

"THE use, which we can make in philosophy, of mathematics," Kant wrote in the year 1763 in the Preface of his *Attempt to introduce the Concept of Negative Magnitudes into Philosophy*, "consists either in the imitation of its methods or in the real application of its propositions to the objects of philosophy. It is not evident that the first has to date been of much use, however much advantage was originally promised from it. The second use, on the contrary, has been so much the more advantageous for the parts of philosophy concerned, which, by the fact that they applied the doctrines of mathematics for their purposes, have raised themselves to a height to which otherwise they could make no claim. These, however, are only doctrines belonging to the theory of nature. . . . As far as metaphysics is concerned, this science, instead of utilizing a few of the concepts or doctrines of mathematics, has rather often armed itself against them and, where it might perhaps have borrowed a sure foundation for its considerations, we see it concerned with making out of the concepts of the mathematician nothing but fine imaginings, which beyond his field have little truth in them. One can easily

decide where the advantage will fall in the conflict of two sciences, of which the one surpasses all others in certainty and clarity, the other of which, however, is only striving to attain certainty and clarity. Metaphysics seeks, *e. g.*, to discover the nature of space and the supreme ground from which its possibility can be understood. Now nothing can be more helpful for this than if one can borrow from somewhere sufficiently proved data to take as a basis for one's consideration. Geometry offers several data, which concern the most general properties of space, *e. g.*, that space does not consist of simple parts; but these are passed by and one sets his trust merely on the ambiguous consciousness of the concept, which is conceived in a wholly abstract fashion. . . . The mathematical consideration of motion in connection with knowledge of space furnishes many data to guide the metaphysical speculations of the times in the track of truth. The celebrated Euler, among others, has given some opportunity for this, but it seems more comfortable to remain with obscure abstractions, which are hard to test, than to enter into connection with a science which possesses only intelligible and obvious insights."

The essay of Euler, to which Kant here refers the metaphysician, is the former's *Réflexions sur l'espace et le temps,* which appeared in the year 1748 among the productions of the Berlin Academy of Science. This essay sets up in fact not only a program for the construction of mechanics but a general program for the epistemology of the natural sciences. It seeks to define the concept of truth of mathematical physics and contrasts it with the concept of truth of the metaphysician. Materially, however, the considerations of Euler rest entirely on the foundations on which Newton had erected the classical system of mechanics. Newton's concepts of absolute space and absolute time are here to be revealed not only as the neces-

sary fundamental concepts of mathematico-physical knowledge of nature, but as true physical realities. To deny these realities on philosophical, on general epistemological grounds means, as Euler explains, to deprive the fundamental laws of dynamics—above all the law of inertia—of any real physical significance. In such an alternative, however, the outcome cannot be questioned: the philosopher must withdraw his suspicions concerning the "possibility" of an absolute space and an absolute time as soon as the reality of both can be shown to be an immediate consequence of the validity of the fundamental laws of motion. What these laws *demand*, also "is"—and it is, it exists in the highest sense and highest degree of objectivity, which is attainable for our knowledge. For before the reality of nature as it is represented in motion and its empirical laws all logical doubt must be silent; it is the business of thought to accept the existence of motion and its fundamental rules instead of attempting to prescribe to nature itself from abstract considerations concerning what can or cannot be conceived.

This demand, however, illuminating as it appears and fruitful as the methodic stimulus of Euler proved in the development of the Kantian problem,[1] becomes problematical when considered from the standpoint of modern physics and epistemology. Kant believed that he possessed in Newton's fundamental work, in the *Philosophiae Naturalis Principia Mathematica,* a fixed code of physical "truth" and believed that he could definitively ground philosophical knowledge on the *"factum"* of mathematical natural science as he here found it; but the relation between philosophy and exact science has since changed fundamentally. Ever more clearly, ever more compellingly do we realize today that the Archimedean point on which he supported himself and from which he undertook to raise the

[1] For more detail concerning Euler and Kant's relation to him, *cf.*, *Erkenntnisproblem* (7), II, 472ff., 698, 703f.

whole system of knowledge, as if by a lever, no longer offers an unconditionally fixed foothold. The *factum* of geometry has lost its unambiguous definiteness; instead of the one geometry of Euclid, we find ourselves facing a plurality of equally justified geometrical systems, which all claim for themselves the same intellectual necessity, and which, as the example of the general theory of relativity seems to show, can rival the system of classical geometry in their applications, in their fruitfulness for physics. And the system of classical mechanics has undergone an even greater transformation, since in modern physics the "mechanical" view of the world has been more and more superseded and replaced by the electro-dynamic view. The laws, which Newton and Euler regarded as the wholly assured and impregnable possession of physical knowledge, those laws in which they believed to be defined the concept of the corporeal world, of matter and motion, in short, of nature itself, appear to us today to be only abstractions by which, at most, we can master a certain region, a definitely limited part of being, and describe it theoretically in a first approximation. And if we turn to contemporary physics with the old philosophical question as to the "essence" of space and time, we receive from it precisely the opposite answer to that which Euler gave the question a hundred and fifty years ago. Newton's concepts of absolute space and absolute time may still count many adherents among the "philosophers," but they seem definitively removed from the methodic and empirical foundations of physics. The general theory of relativity seems herein to be only the ultimate consequence of an intellectual movement, which receives its decisive motives equally from epistemological and physical considerations.

The working together of the two points of view has always come to light with special distinctness at the decisive turning points in the evolution of theoretical

physics. A glance at the history of physics shows that precisely its most weighty and fundamental achievements stand in closest connection with considerations of a general epistemological nature. Galilei's *Dialogues on the Two Systems of the World* are filled with such considerations and his Aristotelian opponents could urge against Galilei that he had devoted more years to the study of philosophy than months to the study of physics. Kepler lays the foundation for his work on the movement of Mars and for his chief work on the harmony of the world in his *Apology for Tycho,* in which he gives a complete methodological account of hypotheses and their various fundamental forms; an account by which he really created the modern concept of physical *theory* and gave it a definite concrete content. Newton also, in the midst of his considerations on the structure of the world, comes back to the most general norms of physical knowledge, to the *regulae philosophandi.* In more recent times, Helmholtz introduces his work, *Uber der Erhaltung der Kraft* (1847), with a consideration of the causal principle as the universal presupposition of all "comprehensibility of nature," and Heinrich Hertz expressly asserts in the preface of his *Prinzipien der Mechanik* (1894), that what is new in the work and what alone he values is 'the order and arrangement of the whole, thus the logical, or, if one will, the philosophical side of the subject."[2] But all these great historical examples of the real inner connection between epistemological problems and physical problems are almost outdone by the way in which this connection has been verified in the foundation of the theory of relativity. Einstein himself—especially in the transition from the special to the general theory of relativity—appeals primarily to an epistemological motive, to which he grants, along with the purely empirical and physical grounds, a decisive

[2] *Cf.* Helmholtz (29, p. 4); H. Hertz (31, p. XXVII).

significance.[3] And even the special theory of relativity is such that its advantage over other explanations, such as Lorentz's hypothesis of contraction, is based not so much on its empirical material as on its pure logical form, not so much on its physical as on its general *systematic* value.[4] In this connection the comparison holds, which Planck has drawn between the theory of relativity and the Copernican cosmological reform. The Copernican view could point, when it appeared, to no single new "fact" by which it was absolutely demanded to the exclusion of all earlier astronomical explanations, but its value and real cogency lay in the fundamental and systematic clarity, which it spread over the whole of the knowledge of nature. In the same way, the theory of relativity, taking its start in a criticism of the concept of time, extends into the field of epistomological problems not merely in its applications and consequences but even in its first beginnings. That the sciences, in particular, mathematics and the exact natural sciences, furnish the criticism of knowledge with its essential material is scarcely questioned after Kant; but here this material is offered to philosophy in a form, which, even of itself, involves a certain epistemological interpretation and treatment.

Thus, the theory of relativity, as opposed to the classical system of mechanics, offers a new scientific problem by which the critical philosophy must be tested anew. If Kant—as Hermann Cohen's works on Kant urged repeatedly and proved from all angles—intended to be the philosophical systematizer of the Newtonian natural science, is not his doctrine necessarily entangled in the fate of the Newtonian physics, and must not all changes in the latter react directly on the form of the fundamental doctrines of the critical philosophy? Or do the doctrines of the Transcendental Aesthetic offer a foundation, which is broad

[3] *Cf.* Einstein (17, p. 8).
[4] See below, Sect., II.

enough and strong enough to bear, along with the structure of the Newtonian mechanics, also that of modern physics? The future development of the criticism of knowledge will depend on the answer to these questions. If it is shown that the modern physical views of space and time lead in the end as far beyond Kant as they do beyond Newton, then the time would have come when, on the basis of Kant's presuppositions, we would have to advance beyond Kant. For the purpose of the *Critique of Pure Reason* was not to ground philosophical knowledge once for all in a fixed dogmatic system of concepts, but to open up for it the "continuous development of a science" in which there can be only relative, not absolute, stopping-points.

Epistemology, however, closely as its own fate is connected with the progress of exact science, must face the problems which are presented to it by the latter, with complete methodic independence. It stands to physics in precisely the relation, in which, according to the Kantian account, the "understanding" stands to experience and nature: it must approach nature "in order to be taught by it: but not in the character of a pupil, who agrees to everything the master likes, but as an appointed judge, who compels the witnesses to answer the questions which he himself proposes." Each answer, which physics imparts concerning the character and the peculiar nature of its fundamental concepts, assumes inevitably for epistemology the form of a question. When, for example, Einstein gives as the essential result of his theory that by it "the last remainder of physical objectivity" is taken from space and time (17, p. 13), this answer of the physicist contains for the epistemologist the precise formulation of his real problem. What are we to understand by the physical objectivity, which is here denied to the concepts of space and time? To the physicist physical objectivity may appear as

a fixed and sure starting-point and as an entirely definite standard of comparison; epistemology must ask that its meaning, that what is to be expressed by it, be exactly defined. For epistemological reflection leads us everywhere to the insight that what the various sciences call the "object" is nothing given in itself, fixed once for all, but that it is first determined by some standpoint of knowledge. According to the changes of this ideal standpoint, there arise for thought various classes and various systems of objects. It is thus always necesary to recognize, in what the individual sciences offer us as their objects and "things," the specific logical conditions on the ground of which they were established. Each science *has* its object only by the fact that it *selects* it from the uniform mass of the given by certain formal concepts, which are peculiar to it. The object of mathematics is different from that of mechanics, the object of abstract mechanics different from that of physics, *etc.*, because there are contained in all these sciences different questions of knowledge, different ways of referring the manifold to the unity of a concept and ordering and mastering the manifold by it. Thus the content of each particular field of knowledge is determined by the characteristic form of judgment and question from which knowledge proceeds. In the form of judgment and question the particular special axioms, by which the sciences are distinguished from each other, are first defined. If we attempt to gain a definite explanation of the concept of "physical objectivity" from this standpoint, we are first led to a negative feature. Whatever this objectivity may mean, in no case can it coincide with what the naïve view of the world is accustomed to regard as the reality of things, as the reality of objects of sensuous perception. For the objects, of which scientific physics treats and for which it establishes its laws, are distinguished from this reality by their general fundamental

form. That concepts, such as those of mass and force, the atom or the ether, the magnetic or electrical potential, even concepts, like those of pressure or of temperature, are no simple thing-concepts, no copies of particular contents given in perception: this scarcely needs any further explanation, after all that the epistemology of physics itself has established concerning the meaning and origin of these concepts. What we possess in them are obviously not reproductions of simple things or sensations, but theoretical assumptions and constructions, which are intended to transform the merely sensible into something measurable, and thus into an "object of physics," that is, into an object *for* physics. Planck's neat formulation of the physical criterion of objectivity, that everything *that can be measured* exists, may appear completely sufficient from the standpoint of physics; from the standpoint of epistemology, it involves the problem of discovering the fundamental conditions of this measurability and of developing them in systematic completeness. For any, even the simplest, measurement must rest on certain theoretical presuppositions on certain "principles," "hypotheses," or "axioms," which it does not take from the world of sense, but which it brings to this world as postulates of thought. In this sense, the reality of the physicist stands over against the reality of immediate perception a something through and through mediated; as a system, not of existing things or properties, but of abstract intellectual symbols, which serve to express certain relations of magnitude and measure, certain functional coördinations and dependencies of phenomena. If we start from this general insight, which within physics itself has been made very clear, especially by Duhem's analysis of the physical construction of concepts, the problem of the theory of relativity gains its full logical definiteness. That physical objectivity is denied to space and time by this theory must, as is now seen, mean

something else and something deeper than the knowledge that the two are not things in the sense of "naïve realism." For things of this sort, we must have left behind us at the threshold of exact scientific physics, in the formulation of its first judgments and propositions. The property of not being thing-concepts, but pure concepts of measurement, space and time share with all other genuine physical concepts; if, in contrast to these, space and time are also to have a special logical position, it must be shown that they are removed in the same direction as these, a step further from the ordinary thing-concepts, and that they thus represent, to a certain extent, concepts and forms of measurement of an order higher than the first order.

The fact appears even in the first considerations, from which the theory of relativity starts, that the physicist does not have only to hold in mind the measured object itself, but also always the particular conditions of measurement. The theory distinguishes between physical determinations and judgments, which result from measurement from resting and moving systems of reference, and it emphasizes the fact that before determinations, which have been gained from diverse systems of reference, can be compared with each other, a universal methodic principle of transformation and permutation must be given. To each objective measurement, there must be added a certain subjective index, which makes known its particular conditions and only when this has taken place can it be used along with others in the scientific construction of the total picture of reality, in the determination of the laws of nature, and be combined with these others into a unitary result. What is gained by this reflection on the conditions of physical measurement in a pure epistemological regard appears as soon as one remembers the conflicts, which have resulted from the lack of this reflection in the course of the history of philosophy and of exact science.

It seems almost the unavoidable fate of the scientific approach to the world that each new and fruitful concept of measurement, which it gains and establishes, should be transformed at once into a thing-concept. Ever again does it believe the truth and the meaning of the physical concepts of magnitude to be assured only when it permits certain absolute realities to correspond to them. Each creative epoch of physics discovers and formulates new characteristic measures for the totality of being and natural process, but each stands in danger of taking these preliminary and relative measures, these temporarily ultimate intellectual instruments of measurement, as definitive expressions of the ontologically real. The history of the concept of matter, of the atom, of the concepts of the ether and of energy offer the typical proof and examples of this. All materialism—and there is a materialism not only of "matter" but also of force, of energy, of the ether, *etc.*, —goes back from the standpoint of epistemology, to this one motive. The ultimate constants of physical calculation are not only taken as real, but they are ultimately raised to the rank of that which is alone real. The development of idealistic philosophy itself is not able to escape this tendency. Descartes as an idealistic mathematician was at the same time the founder of the "mechanical view of the world." Since only extension offers us exact and distinct concepts and since all clearly comprehended truth is also the truth of the *existing*, it follows, in his view, that mathematics and nature, the system of measurements and the totality of material existence, must be identified. The manner, in which the same step from the logico-mathematical to the ontological concept has been repeated in the development of modern energetics, is known. Here, after energy had been discovered as a fundamental measure, as a measure which is not limited to the phenomena of motion, but spans equally all physical fields, it was made an all-

inclusive substance, which rivalled "matter" and finally took it up into itself completely. But on the whole, we are here concerned only with a metaphysical by-way, which has not seduced science itself from its sure methodic course. For the concept of energy belongs in conception to that general direction of physical thought, which has been called the "physics of principles" in contrast to the physics of pictures and mechanical models. A "principle," however, is never directly related to things and relations of things, but is meant to establish a general rule for complex functional dependencies and their reciprocal connection. This rule proves to be the really permanent and substantial: the epistemological, as well as the physical, value of energetics is not founded on a new pictorial representation to be substituted for the old concepts of "matter" and "force" but on the gaining of *equivalence-numbers,* such as were expressly demanded and discovered by Robert Mayer as the "foundation of exact investigation of nature." (*Cf.* 52, p. 145, 237ff.)

Even in these two examples we can learn that through the whole history of physics there is a certain intellectual movement, which throughout runs parallel to the movement in epistemology that mediates and passes to and fro between the "subject" and the "object" of knowledge. Physical thought is always concerned at first with establishing a characteristic standard of measurement in an objective physical concept, in a certain natural constant. Then it is concerned, in the further development, with understanding more and more clearly the constructive element that is contained in any such original constant, and with becoming conscious of its own conditionality. For, whatever particular properties they may have, no constants are immediately given, but all must be *conceived* and *sought* before they can be found in experience. One of the most pregnant examples of this is found in the his-

tory of the concept of the atom. The atoms were postulated by Democritus as ultimate constants of nature long before thought possessed any means of concretely realizing this postulate. Fundamentally, such a realization, such a strictly quantitative meaning of the concept of the atom, was only reached in the beginnings of modern chemistry in the law of multiple proportion. To the extent, however, that to this particular realization of the concept of the atom in the law of multiple proportion others and still others are added and the concept of the atom finally comes to characterize and to organize intellectually the most diverse fields, its character as a pure principle, which was originally fused with its thing-character, comes to light. The content of the idea of the atom changes and shifts from place to place in the course of the development of physics and chemistry, but the function of the atom as the temporarily ultimate unit of measurement remains. When we pass from the consideration of "ponderable" matter to the consideration of the ether, when we seek a unity, which comprehends not only the mechanical but also the optical and electrical phenomena, the atom of matter becomes the atom of electricity, the electron. In recent physics, there appears further, with Planck's Quanta Theory the thought of an atomistic structure not only of matter but of energy. It would be in vain were one to attempt to combine all these various applications of the concept of the atom in chemistry, in the kinetic theory of gases, and in the doctrine of light and heat radiation, *etc.*, into a unitary picture. But the unity of its meaning requires no such pictorial unity; it is satisfied, indeed verified in a far stricter logical sense, when it is shown that here a common relation, a peculiar "form" of connection, prevails, which as such can be verified and represented in the most diverse contents. The atom shows itself thereby to be, not an absolute minimum of being, but a relative mini-

mum of measure. It was one of the founders of modern philosophy, Nicholas Cusanus, who, with true speculative profundity, anticipated and announced this as the function of the concept of the atom, which was to be actually realized only in the history of natural science. Cusanus' fundamental doctrine of the infinite and of the unity of opposites in the infinite rested entirely on this insight into the relativity in principle of all determinations of magnitude, on the coincidence of the "greatest" and the "smallest." (*Cf.* 7, I, 40ff, 265ff.) Modern criticism of knowledge brings the riddle, with which Cusanus' doctrine of the minimum struggles, to a simple expression. Contradiction only enters when we attempt to unify after the fashion of a thing all the different forms, which the thought of the "smallest" assumes, in the different fields of thought; but it disappears as soon as we reflect that the true unity is never to be sought in things as such, but in intellectual constructions, which we choose according to the peculiarity of the field to be measured, and which are thus in principle possessed of an unlimited variability. It follows from this that, as *what is to be measured* is unlimited in variety, so *what measures* can be represented in infinitely many and infinitely diverse ways. In other words, the unity that we have to seek lies neither in the one nor the other member, but merely in the form of their reciprocal connection, *i. e.,* in the logical conditions of the operation of measurement itself.

This receives new confirmation when we pass from the concept of matter, of energy and of the atom to the real concept of objectivity of modern physics, that of motion. The historical beginnings of the modern theory of motion in Galilei refer directly to the epistemological question, which has received its definitive formulation in the general theory of relativity. What Galilei gained with *his* idea of relativity was the cancelling of the absolute reality of

place, and this first step involved for him the most weighty logical consequences, viz., the new concept of the lawfulness of nature and the new interpretation of the particular laws of dynamics. Galilei's doctrine of motion is rooted in nothing less and nothing more than in the choice of a new standpoint from which to estimate and measure the phenomena of motion in the universe. By this choice, there was given him at once the law of inertia and in it the real foundation of the new view of nature. The ancient view saw in place a certain physical property that produced definite physical effects. The "here" and "there," the "above" and "below," were for it no mere relations; but the particular point of space was taken as an independent real, which consequently was provided with particular forces. In the striving of bodies to their "natural places," in the pressure of air and fire upwards and in the sinking of heavy masses downwards, these forces seemed given as immediate empirical realities. Only when one takes account of these fundamental features, not only of ancient astronomy and cosmology but also of ancient physics, does one understand the whole boldness of the new intellectual orientation, resulting from the Copernican system of the world. One of the most fixed and certain realities on which Grecian thought had constructed its picture of the world now became a mere illusion, a purely "subjective" feature. Even the first adherents of the new doctrine drew the decisive conclusion with reference to the doctrine of place. What Gilbert, *e. g.*, urges against the Aristotelian physics and cosmology is above all this epistemological feature, *i. e.*, that it permits the ideal and the real to flow into each other. Differences belonging merely to our thought, to our subjective reflection, are throughout made into objective oppositions. But in truth no place in itself is opposed to any other, but there are in nature only differences in the mutual positions of bodies and of material masses. "It

is not place which, in the nature of things, works and produces, which determines the rest and motion of bodies. For it is in itself neither a being nor an effective cause; rather bodies determine their mutual place and position by virtue of the forces which are immanent in them. The place is a nothing; it does not exist and exerts no force, but all natural power is contained and grounded in bodies themselves." (7, I, 360f.) It is implied in this that what we call the "true place" is never given to us as an immediate sensuous property, but must be discovered on the basis of calculation and of the "arithmetic of forces" in the universe. All determination of place—as Kepler sharply and clearly expresses this insight which for him resulted equally from astronomical convictions, physiological optics and analysis of the general problem of perception—is a work of the mind: *omnis locatio mentis est opus.* (37, II, 55, *cf.* 7, I, 339.) From this point, the way is open to Galilei's foundation of dynamics: for since place has ceased to be something real, the question as to the *ground* of the place of a body and the ground of its *persistence* in one and the same place disappears. Objective physical reality passes from *place* to *change of place,* to motion and the factors by which it is determined as a magnitude. If such a determination is to be possible in a definite way, the identity and permanence, which were hitherto ascribed to mere places, must go over to motion; motion must possess "being," that is, from the standpoint of the physicist, numerical constancy. This demand for the numerical constancy of motion itself finds its expression and its realization in the law of inertia. We recognize here again how closely, in Galilei, the mathematical motive of his thought was connected with an ontological motive, how his conception of *being* interacted with his conception of *measure.* The new measure, which is found in inertia and in the concept of uniform acceleration, in-

volves also a new determination of reality. In contrast with mere place, which is infinitely ambiguous and differs according to the choice of the system of reference, the inertial movement appears to be a truly intrinsic property of bodies, which belongs to them "in themselves" and without reference to a definite system of comparison and measurement. The velocity of a material system is more than a mere factor for calculation; it not only really belongs to the system but defines its reality since it determines its *vis viva, i. e.,* the measure of its dynamic effectiveness. In its measure of motion, in the differential quotient of the space by the time, Galilei's physics claims to have reached the kernel of all physical being, to have defined the intensive reality of motion. By this reality, the dynamic consideration is distinguished from the merely phoronomic. The concept of the "state of motion," not as a mere comparative magnitude, but as an essential element belonging to the moving system intrinsically, now becomes the real mark and characteristic of physical reality. Leibniz, too, in his foundation of dynamics, stands throughout at this standpoint, which becomes for him a starting-point for a new metaphysics of forces. Motion conceived as a mere change of place in the purely phoronomical sense, he explains, remains always something purely relative; it only becomes an expression of a true physical and metaphysical reality when we add to it an inner dynamic principle, a force conceived as an "originally implanted principle of permanence and change," *principium mutationis et perseverantiae.* (42, VI, 100 *cf.* 5, p. 290ff.) In all these examples, it is evident how sharply, on the one hand the physical thought of modern times has grasped the thought of the relativity of place and of motion, and, on the other hand, how it has shrunk back from following it to its ultimate consequences. If not only place but the velocity of a material system is to

signify a magnitude that entirely depends on the choice of the reference body and is thus infinitely variable and infinitely ambiguous, there seems no possibility of an exact determination of magnitude and thus no possibility of an exact objective determination of the state of physical reality. Pure mathematics may be constructed as the ideal doctrine of the comparison and connection of magnitudes, as a system of mere relations and functions and may come to recognize itself as such ever more clearly, but physics seems necessarily to reach an ultimate limit, a *non plus ultra*, if it is not wholly to lose any basis in reality.

The difficulty, which remains in the structure of classical mechanics in the formulation of the principles of inertia, is expressed in an epistemological circle, from which there seems no escape. To understand the meaning of the law of inertia, we need the concept of "equal times" but a practicable physical measure of equal times can, as is discovered, only be gained by presupposing, in its content and validity, the law of inertia. In fact, since Carl Neumann's work, *Uber die Prinzipien der Galilei-Newton'schen Theorie* (57), which set in motion the modern discussion on the law of inertia, it is customary in mechanics to define equal times as times within which a body left to itself traverses equal distances. Maxwell, too, in his exposition of the Newtonian mechanics, conceived the law of inertia as a pure definition of measure. The first law of Newton, as he explains clearly and pregnantly, tells under what conditions no external force is present. (51, p. 31.) Thus in the progress of mechanics the principle of inertia is *recognized* with increasing distinctness as what it *meant* fundamentally to Galilei. It is no longer taken as a direct empirical description of given processes of nature, but as the "axiom of the field," the fundamental hypothesis by which the new science of dynamics prescribes to itself a certain form of measurement. Inertia appears, not as

an absolute and inherent property of things and of bodies, but as the free establishment of a certain standard and symbol of measurement, by virtue of which we can hope to reach a systematic conception of the laws of motion. In this alone is rooted its reality, *i. e.,* its objective and physical significance. Thus, within the historical development of physics itself *what measures* is separated with increasing distinctness from *what is measured,* with which it at first seems to coincide; the observable data of experience are separated with increasing distinctness from what must be presupposed and used as a *condition* of observation and of measurement.

And what is here seen in a particular example and within a narrow field is repeated, on closer examination, in all the special fields of physics. Everywhere physical thought must determine for itself its own standards of measurement before it proceeds to observation. There must be established a certain standpoint for the comparison and correlation of magnitudes; certain constants must be established at least hypothetically and in preliminary fashion before a concrete measurement can take place. In this sense, each measurement contains a purely ideal element; it is not so much with the sensuous instruments of measurement that we measure natural processes as with our own thoughts. The instruments of measurement are, as it were, only the visible embodiments of these thoughts, for each of them involves its own theory and offers correct and useful results only in so far as this theory is assumed to be valid. (*Cf.* 8, p. 189ff.). It is not clocks and physical measuring-rods but principles and postulates that are the real instruments of measurement. For in the multiplicity and mutability of natural phenomena, thought *possesses* a relatively fixed standpoint only by *taking* it. In the choice of this standpoint, however, it is not absolutely determined by the phenomena, but the choice re-

mains its own deed for which ultimately it alone is responsible. The decision is made with reference to experience, *i. e.,* to the connection of observations according to law, but it is not prescribed in a definite way by the mere sum of observations. For these in themselves can always be expressed by a number of intellectual approaches between which a choice is possible only with reference to logical "simplicity," more exactly, to systematic unity and completeness, of scientific exposition. When thought, in accordance with its claims and demands, changes the form of the "simple" fundamental measuring relations, we stand before a new "picture" of the world with regard to content also. The previously gained relations of experience do not indeed lose their validity, but, since they are expressed in a new conceptual language, they enter into a new system of meanings. The fixed Archimedean point of the former view of the world moves; the previous ποῦ στῶ of thought appears transcended. But it is soon seen that thought, by virtue of its peculiar function, can only transcend an earlier construction by replacing it by a more general and more inclusive one; that it only shifts, among phenomena, the constancy and identity, which it cannot cease to demand, to another and deeper place. That every realization, which the demand of thought for ultimate constants can find within the empirical world is always only conditioned and relative, is guaranteed by the unconditionality and radicalism of precisely this demand. The critical theory of knowledge would not only show this connection *in abstracto,* but for it the concrete movement of thought, the continual oscillation between experience and concept, between facts and hypotheses in the history of physics, forms a perpetually new source of instruction. In the midst of the change of particular theoretical instruments of measurement, the critical theory holds fast to the thought of the unity of measurement,

which indeed signifies for it no realistic dogma but an ideal goal and a never-to-be-concluded task. Each new physical hypothesis erects, as it were, a new logical system of coördinates, to which we refer phenomena, while nevertheless the doctrine is retained as a regulative idea for investigation that all these systems converge on a certain definite limiting value. In the confusion and continuous flux of phenomena, the understanding seems at first almost arbitrarily to fix and separate out certain points in order to learn through them a definite law of change, but everything which it regards as determined and valid in this sense proves, in the course of further progress, to be a mere approximation. The first construction must be both limited and more exactly defined logically by the second, this again by the third, *etc.* Thus, ever anew does the temporarily chosen theoretical center of thought shift; but in this process, the sphere of being, the sphere of objective knowledge, is more and more penetrated by thought. As often as it seems that thought is overturned by new facts and observations, which are outside its previously formulated laws, it is seen that, in fact, thought has found in them a new point of leverage, around which moves henceforth the totality of empirically provable "facts." The epistemological exposition and evaluation of each new physical theory must always seek to indicate the ideal center and turning-point around which it causes the totality of phenomena, the real and possible observations, to revolve,—whether this point is clearly marked or whether the theory only refers to it indirectly by the intellectual tendency of all its propositions and deductions.

II

The Empirical and Conceptual Foundations of the Theory of Relativity

If there can be no doubt, according to the opening words of the *Critique of Pure Reason,* that all our knowledge begins with experience, then this holds especially when we are concerned with the origin of a physical theory. The question here can never run as to *whether* the theory has issued from experience but merely as to *how* it is based on experience and what is the relation of the diverse elements which characterize and make up the concept of experience as such. There is accordingly needed no special epistemological analysis to make clear the relation of the special and general theories of relativity to experience, to the whole of observation and of physical experiment; such an analysis will only have to decide whether the theory in its origin and development is to be taken as an example and witness of the critical or of the sensualistic concept of experience. Does "experience," as it is used here, mean merely the bare sum of particular observations—*experimentorum multorum coacervatio,* as a sensualistic thinker once described it—or is there involved in it an independent intellectual form? Is the construction of the theory merely a matter of joining "fact" to "fact," perception to perception,—or, in this connection of particulars, have there been effective all along certain universal and critical norms, certain methodic presuppositions? No "empiricism" however extreme can ever seek to deny the rôle of thought in establishing and grounding physical the-

ories, and just as little is there, on the other hand, a logical idealism, which would attempt to free "pure thought" from reference to the world of the "factual" and from being bound to it. The question dividing the two views can only be as to whether thought consists in a simple registration of facts, or whether, even in the *establishment,* in gaining and interpretation of "particular facts," thought reveals its characteristic power and function. Is its work completed in arranging particular data, immediately taken from sense perception, like pearls on a thread—or does it face them with its own original measures, as independent criteria of judgment?

The problem raised here received its first sharp and clear systematic formulation in the Platonic doctrine of ideas. For Platonic idealism, too, the proposition holds that it is not possible to think save on the basis of some perception: οὐ δυνατὸν ἐννοεῖν ἢ ἔκ τινος αἰσθήσεως. But the function of the "logic in us" consists indeed not in finding the sum of the particular perceptions, not in deriving and deducing the "idea of the equal" from the "equal pieces of wood and stone," but the "logic in us" is revealed in discriminating and judging what is given in perception. This discriminating constitutes the real fundamental character of thought as διάνοια, as *discursus.* Not all perceptions and observations stimulate equally the critical and discriminating activity of thought. There are some which do not summon the understanding to reflection since satisfaction is done them by mere sensation, but there are others which in all ways call forth thought as if in their case perception by itself could gain nothing solid. "Not exciting, namely, is that which does not pass into an opposite perception; exciting objects I call those which give opposite perceptions, because here perception gives no more vivid idea of any particular object than of its opposite. Much in perception is a paraclete of thought (παρακλητικὰ

τῆς διανοίας), while other perceptions are not—*such an awakener of thought, namely, is everything, which comes into sense at the same time as its opposite;* but what does not, that also does not arouse thought." (*Republic* 523-524.) In this Platonic characterization of the relation of thought and sensation, of reason and sensibility, we have, as Cohen has urged, "one of the most fundamental thoughts in the evolution of the critique of cognition." (12, p. 16ff.) Just as for Plato thought becomes what it is in assertion and contradiction, in dialectic, so not any arbitrary perception but only one to which this feature corresponds can become its awakener and paraclete. The dialectic of perception summons that of thought to judgment and decision. Where the perceptions, as it were, rest peacefully side by side, where there is no inner tension between them, thought rests also; only where they contradict each other, where they threaten to cancel each other does thought's fundamental postulate, its unconditional demand for unity stand forth and demand a transformation, a reshaping of experience itself.

The evolution of the theory of relativity has furnished a new typical proof of this general relation. It was in fact a fundamental contradiction between physical experiments from which the theory of relativity took its start. On the one side stood the investigation of Fizeau, on the other, that of Michelson, and the two seemed in their results absolutely irreconcilable. Both sought to gain an answer to the question as to how the velocity of light in a moving medium was related to its velocity in a resting medium; and they answered this question in completely opposite ways. The investigation of Fizeau showed that the velocity of light in flowing water was greater than in water at rest; that, however, not the whole velocity of the flowing water, but only a fraction of it was added to the velocity of light in a medium at rest. If we call the

velocity of light in the moving medium W and the velocity of light in a medium at rest w and the velocity of the flowing v, it results not simply that $W=w+v$, but rather that $W=w+v(1-\frac{1}{n^2})$, in which the magnitude $n=\frac{c}{w}$ signifies the exponent of the refraction of the liquid. This result, as interpreted by the theory of Lorentz, spoke directly for the assumption of a motionless ether not carried along by the body in its movement. But the attempt of Michelson, to discover the consequences of the movement of the earth with reference to this motionless ether, failed. In no way could any influence be shown of the motion of the earth on the velocity of the propagation of light; it was rather shown with increasing evidence that all optical phenomena take place as if there were no translation of the earth against the ether.[5] And behind this conflict of "facts" there stood, as one was forced to recognize more and more, a conflict in general principles, to which the theories of mechanical and of optical and electromagnetic phenomena seemed to lead necessarily. Experiments in the latter could finally be summarized in a single proposition, the principle of the constancy of the velocity of light in a vacuum. The validity of the fundamental equations of electro-dynamics of Maxwell and Hertz involved the assumption that light in an empty space is always propagated with a definite velocity V independently of the state of motion of the body emitting it. From whatever system one made the observation and from whatever source the light issued, there would always be found the same determinate value for its velocity of propagation. But this assumption of the velocity of light as a universal constant the same for all systems, necessarily demanded by the principles of electro-dynamics, now

[5] For more detail concerning the investigations of Fizeau and Michelson as well as concerning the negative outcome of other investigations on the influence of the movement of the earth on optical and electrical phenomena, *cf.* Laue (40), p. 10ff.

comes into opposition with the principle of relativity of the Galilei-Newtonian mechanics. This principle demands that, when any definite Galileian reference body is given —*i. e.,* a body relatively to which a body "left to itself" persists in its state of rest or of uniform motion in a straight line—all the laws, which are valid relatively to this reference body K remain valid when one passes to the system of reference K′, which is, with reference to K, in uniform translatory motion. In the transition from K to K′, the equations of the "Galilei-transformation" hold,

$$x'=x-vt,\ y'=y,\ z'=z$$

(where v signifies a constant velocity of K′ with reference to K parallel to the x and x′ axes), to which there is to be added the identical transformation from the time $t'=t$, which is not especially noticed in classical mechanics. If we seek, however, to apply the principle of relativity of mechanics to electro-dynamics, *i. e.,* to recalculate its equations according to the formulae of the Galilei-transformation, it is seen that this cannot be done: the electro-dynamic equations, in contrast to the Newtonian equations of motion, alter their form when we insert the coördinates x′, y′, z′, t′, into them in place of the coördinates x, y, z, t according to the rules of the Galilei-transformation. The effort to unite mechanics and electro-dynamics by carrying over the principle of relativity of the first into the latter thus had to be given up: the Hertzian theory, which represented such an attempt, came into irreconcilable conflict with assured experimental results. Physical investigation stood before the dilemma of giving up a principle which had been verified without exception in all the phenomena of motion and which formed a corner-stone in the structure of classical mechanics—or of retaining it within its field but denying its applicability to optical and electromagnetic phenomena. In both cases, the unity of the explanation of nature, the unity of the very concept of nature,

seemed destroyed. Here in fact the condition set up by Plato, of the intellectual fruitfulness of experience was fulfilled: here experience stood at a point at which assured observation seemed to pass directly into its opposite. The conflict between the principle of the constancy of the propagation of light and the principle of relativity of mechanics became the "paraclete of thought"—the real awakener of the theory of relativity.

But how did physical thought go about overcoming this conflict, since it was bound to the outcome of observation as such, since it could neither set aside the facts expressed in the principle of the constant velocity of light in a vacuum, nor those expressed in the principle of relativity of mechanics? If we look back on the historical development of the theory of relativity, we recognize that the latter has followed here a counsel which was once given by Goethe. "The greatest art in theoretical and practical life," wrote Goethe to Zelter, "consists in changing the *problem* into a *postulate;* that way one succeeds." In fact, this was the course which Einstein followed in his fundamental essay, '*Zur Elektrodynamik bewegter Systeme* of the year 1905. The principle of the constancy of the velocity of light was given first place as a postulate, but,—supported by the negative result of all attempts to establish an "absolute" motion with reference to a chosen system of reference, *i. e.,* the "motionless ether,"—the *supposition* was made that there correspond to the concept of absolute rest no properties of phenomena in either mechanics or electro-dynamics, but rather that the same electro-dynamic and optical laws hold for all systems of coördinates of which the mechanical equation hold. And this "*supposition*" does not continue such, but is expressly "made a presupposition," *i. e.,* a shaping of theory is demanded which will simultaneously satisfy the conditions of the principle of relativity and those of the principle of

the constant propagation of light. (*Cf.* 16, p. 26.) The two assumptions are indeed not compatible according to the means and habits of thought at the disposal of the kinematics generally accepted before the establishment of the theory of relativity, but they—*ought* no longer to be incompatible. The demand made of physical theory was that it remove this incompatibility by subjecting precisely these means and habits of thought themselves to a critical examination. By an analysis of the physical concepts of space and time, it now appears that in fact the incompatibility of the principle of relativity with the law of the propagation of light is not to be found; that rather there is only needed a transformation of these concepts in order to reach a logically unobjectionable theory. The decisive step is taken when it is seen that the measurements, to be gained within a system by definite physical methods of measurement, by the application of fixed measuring-rods and clocks, have no "absolute" meaning fixed once for all, but that they are dependent on the state of motion of the system and must necessarily result differently according to the latter. There now arises the purely mathematical problem of discovering the laws of permutation, according to which the space-time values of an event are changed in going from one reference body to another, which is in uniform translatory motion with regard to the first. This problem is solved, as is known, by the fundamental equations of the "Lorentz-transformation":

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y \qquad\qquad z' = z$$

$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}}$$

On the basis of these equations, we see that the law of the propagation of light in a vacuum is equally fulfilled for

all justified systems K and K′; on the other hand, it is seen that Maxwell's fundamental equations of electrodynamics do not change their form when the formulae of the Lorentz-transformation rather than those of the Galilei-transformation are applied to them. There is thus a universal principle of relativity, which comprehends the totality of physical phenomena; the laws, according to which the states of physical systems change, are independent of whether they are referred to one or the other of two systems of coördinates in uniform translatory motion relative to each other. (*Cf.* 16, p. 29). The principle of relativity of classical mechanics is so little contradicted by this general principle that it is rather contained in it as a special case; the equations of the Galilei-transformation directly issue from those of the Lorentz-transformation when one considers only such velocities v as are very small in comparison with the velocity of light so that the values $\frac{v}{c^2}$ $\frac{v^2}{c^2}$ can practically be left out of account. It follows from this that the principle of relativity of electrodynamics, carried over to mechanics, can come into conflict with no empirical result, while the converse carrying-over of the principle of relativity of mechanics to electrodynamics proves to be impossible, as the collapse of Hertz's theory showed. More closely considered, however, in the special theory of relativity, the electrodynamic processes are not used as a key to the mechanical, but a truly universal principle, a heuristic maxim of investigation in general, is established, which claims to contain a criterion of the validity and permissibility of all particular physical fields and theories. Thus it is seen that the initial contradiction, appearing between the principles of mechanics and those of electrodynamics, has shown the way to a far more perfect and deeper unity between them than previously existed. And this result was not reached entirely by heaping up experiments, by newly instituted investigations, but it

rests on a critical transformation of the system of fundamental physical concepts.

On the purely epistemological side, there thus appears with special distinctness in this intellectual process in which the theory of relativity originates, that peculiar "Copernican revolution," that variation in the conceptual foundations of the theory of nature, which we have previously traced in the example of classical mechanics and the older physics. An essential part of its achievement seems based on the fact, that it has shifted the previous logical constants of physical knowledge, that it has set them at another place than before. For classical mechanics, the fixed and immovable point was the assumption of the identity of the spatial and temporal values gained by measurement in the various systems. This identity was taken to be the unquestionable and sure foundation of the concept of objectivity in general: as that which first really constituted the object of "nature" as a geometrical and mechanical object and distinguished it from the changeable and relative data of sensation. τὸ μὲν σχῆμα καθ' αὑτό ἐστι, τὸ δὲ γλυκὺ καί δλως τὸ αἰσθητὸν πρὸς ἄλλο καὶ ἐν ἄλλονς—thus runs the proposition, which Democritus brought into the foundations of atomism, and which in modern times was taken up by Galilei to support the fundamental distinction between "primary" and "secondary" qualities, and thus the whole "mechanical" view of the world. Although the principle here established proved to be very fruitful and has been frequently confirmed in mathematical physics, the modern evolution of physics shows, with increasing evidence, that it was conceived too narrowly in a philosophical and methodological sense. The true goal of science is not mechanism but unity—as Henri Poincaré once formulated the guiding maxim of modern physics. But concerning this unity the physicist does not need to ask *whether* it is, but merely *how* it is; *i. e.,* what is the minimum of presup-

positions that are necessary and sufficient to provide an exact exposition of the totality of experience and its systematic connection. (72, p. 172ff.) In order to maintain this unity, which seemed endangered by the conflict of the principle of the constancy of the velocity of light and the principle of relativity of mechanics, and to ground it more deeply and securely, the theory of relativity renounces the unity of the values of spatial and temporal magnitudes in different systems. It surrenders the assumption that the temporal interval between events is a magnitude fixed once for all independently of the state of motion of the reference body and that in the same way the spatial distance between two points of a rigid body is independent of the state of motion of the reference body. By going back to the method of measuring time and to the fundamental rôle that the velocity of light plays in all our physical time measurements, it discovers the *relativity of the simultaneity* of two processes and further leads to the insight that the magnitude of the length of a body, of its volume, its form, its energy and temperature, *etc.*, are, as results from the formulae of the Lorentz-transformation, to be assumed as different according to the choice of the system of reference in which measurement takes place. But these "relativizations" are not in contradiction with the doctrine of the constancy and unity of nature; they are rather demanded and worked out in the name of this very unity. The variation of the measurements of space and time constitutes the necessary condition through which the new invariants of the theory are discovered and grounded. Such invariants are found in the equal magnitude of the velocity of light for all systems and further in a series of other magnitudes, such as the entropy of a body, its electrical charge or the mechanical equivalent of heat, which are unchanged by the Lorentz-transformation and which thus possess the same value in all justified systems of refer-

ence. But above all it is the general form of natural law which we have to recognize as the real invariant and thus as the real logical framework of nature in general. While the special theory of relativity limits itself to regarding all reference bodies K′ which are moving uniformly in a straight line relatively to a definite justified reference system K, as equivalent for the formulation of natural laws, the general theory extends this proposition to the assertion that all reference bodies KK′, whatever their state of motion may be, are to be taken as equivalent for the description of nature. (17, p. 9; 18, p. 42.) But the path by which alone this true universality of the concept of nature and of natural law, *i. e.*, a definite and objectively valid description of phenomena independent of the choice of the system of reference, is to be reached, leads, as the theory shows, necessarily through the "relativization" of the spatial and temporal magnitudes, that hold within the individual system; to take these as changeable, as transformable, means to press through to the true invariance of the genuine universal constants of nature and universal laws of nature. The postulate of the constancy of the velocity of light and the postulate of relativity show themselves thus as the two fixed points of the theory, as the resting intellectual poles around which phenomena revolve; and in this it is seen that the previous logical constants of the theory of nature, *i. e.*, the whole system of conceptual and numerical values, hitherto taken as absolutely determinate and fixed, must be set in flux in order to satisfy the new and more strict demand for unity made by physical thought.

Thus reference to experience, regard for phenomena and their unified exposition, proves to be everywhere the fundamental feature, but at the same time it is seen that,

in the words of Goethe, experience is always only half experience; for it is not the mere observational material as such, but the ideal form and the intellectual interpretation, which it is given, that is the basis of the real value of the theory of relativity and of its advantage over other types of explanation. As is known, the investigation of Michelson and Morley, which gave the impetus and starting-point for the development of the theory of relativity, was explained as early as the year 1904 by Lorentz in a manner which fulfilled all purely physical demands. The Lorentzian hypothesis, that each body moving with reference to the motionless ether with a velocity v undergoes a certain shortening in the dimension parallel to the motion, and indeed in the ratio of $1 : \sqrt{1 - \frac{v^2}{c^2}}$, was sufficient to give a complete explanation of all known observations. An experimental decision between Lorentz's and Einstein's theories was thus not possible; it was seen that between them there could fundamentally be no *experimentum crucis.*[6] The advocates of the new doctrine accordingly had to appeal—a strange spectacle in the history of physics—to general philosophical grounds,—to the advantages over the assumption of Lorentz which the new doctrine possessed in a systematic and epistemological respect. "A really experimental decision between the theory of Lorentz and the theory of relativity," Laue, *e. g.*, explains in his exposition of the principle of relativity in the year 1911, "is indeed not to be gained, and that the former, in spite of this, has receded into the background, is chiefly due to the fact, that, close as it comes to the theory of relativity, it still lacks the great simple universal principle, the possession of which lends the theory of relativity . . . an

[6] For more detail *cf. e. g.* Ehrenfest (15a), p. 16ff.

imposing appearance."[7] Lorentz's assumption appeared above all to be epistemologically unsatisfactory because it ascribes to a physical object, the ether, definite effects, while at the same time it results from these effects that ether can never be an object of possible observation. Minkowski too explains in his lecture on space and time that the Lorentzian hypothesis sounds extremely fantastical; for the contraction is not to be conceived as a physical consequence of the resistence of the ether but rather purely as "a gift from above," as an accompaniment of the state of motion. (47, p. 60f.) What thus, in the last analysis, decided against this assumption was not an empirical but a methodological defect. It conflicted most sharply with a general principle, to which Leibniz had appealed in his struggle against the Newtonian concepts of absolute space and time, and which he formulated as the "principle of observability" (*principe de l'observabilité.*) When Clarke, as the representative of Newton, referred to the possibility that the universe in its motion relatively to absolute space might undergo retardation or acceleration which would not be discoverable for our means of measurement, Leibniz answered that nothing fundamentally outside the sphere of observation possessed "being" in the physical sense: *quand il n'y a point de changement observable, il n'y a point de changement du tout* (5, p. 247ff). It is precisely this principle of "observability," which Einstein applied at an important and decisive place in his theory, at the transition from the special to the general theory of relativity, and which he has attempted to give a necessary

[7] 40, p. 19f.; *cf.* 41, p. 106. *Cf.* also the characteristic remark of Lorentz himself in his Haarlem lecture: "The estimation (of the fundamental concepts of Einstein's theory of relativity) *belongs to a very large extent* (*grösstenteils*) *to the theory of knowledge,* and one can leave the judgment to the latter in confidence that it will consider the questions mentioned with the necessary thoroughness. But it is certain that it will depend for a great part on the type of thought to which one is accustomed, whether one feels drawn more to the one or the other conception. As far as concerns the lecturer himself, he finds a certain satisfaction in the older conceptions, that ether possesses at least some substantiality, that space and time can be sharply separated, that one can talk of simultaneity without further specification." (46a, p. 23.)

connection with the general principle of causality. Any physical explanation of a phenomenon, he urges, is *epistemologically* satisfactory only when there enter into it no non-observable elements; for the law of causality is an assertion concerning the world of experience only when *observable facts* occur as causes and effects. (17, § 2). Here we stand before one of the fundamental intellectual motives of the theory of relativity—a motive which not only gives it the advantage over the empirically equivalent hypothesis of Lorentz, but which also produces the advance from the more limited interpretation of the postulate of relativity in the special theory to the completely universal formulation.

The way in which this advance has taken plase is especially suited to make clear the conceptual and empirical presuppositions of the theory and their reciprocal connection. The special theory of relativity rests, as has been shown, on two different assumptions, which stand, equally justified, side by side: on the postulate of the uniformity of the propagation of light in a vacuum and on the presupposition that all reference systems in rectilinear, uniform and non-rotary motion relatively to a definite justified system K are equally permissible for the formulation of the laws of nature. If one considers these presuppositions, which stand in inseparable connection in the empirical structure of the special theory of relativity, from a purely methodological standpoint, it is seen that in *this* respect they belong to different strata. On the one side, stands the assertion of a general fact, a constant of nature, which results from the experimental findings of optics and electrodynamics; on the other side stands a demand, which we make of the *form* of natural laws. In the first case, it is empirically established that there is a peculiar velocity with a definite finite value, which retains this value in any system independently of the state of motion of the latter. In

the second, a general *maxim* is established for the investigation of nature, which is to serve as a "heuristic aid in the *search* for the general laws of nature." In the formal limitation, which is placed on natural laws by this maxim, lies—as Einstein himself has urged—the characteristic "penetration" (*Spürkraft*) of the principle of relativity. (18, pp. 28, 67.) But the two principles, the "material" and the "formal" are not distinguished from each other in the shaping of the special theory of relativity. The fact that this distinction is made and that the general and "formal" principle is placed above the particular and "material" principle constitutes, from the purely epistemological standpoint, the essential step taken by the general theory of relativity. And this step seems to lead to a strange and paradoxical consequence; for the particular result is not taken up into the general, but rather is cancelled by it. From the standpoint of the general theory of relativity, the law of the constancy of the velocity of light in a vacuum no longer possesses unlimited validity. According to the general theory of relativity the velocity of light is dependent on the gravitation potential and must thus in general vary with places. The velocity of light must always depend on the coördinates when a field of gravitation is present; it is only to be regarded as constant when we have in mind regions with a constant gravitation potential. This consequence of the general theory of relativity has often been regarded as a refutation of the presupposition from which the special theory of relativity took its start and on which it based all its deductions. But with justice Einstein rejects any such conclusion. The special theory of relativity, he explains, is not rendered valueless by the fact that one comes to see that its propositions refer to a definitely limited field, namely, to the phenomena in an approximately constant field of gravitation. "Before the establishment of electrodynamics, the laws of electro-

statics were regarded as the laws of electricity in general. Today we know that electrostatics can only describe electrical fields correctly in the case, that is never exactly realized, in which the electric masses are exactly at rest relatively to each other and to the system of coördinates. Is electrostatics overthrown by Maxwell's electrodynamical equations? Not in the least! Electrostatics is contained as a limiting case in electrodynamics; the laws of the latter lead directly to those of the first for the case that the fields are temporarily unchangeable. The most beautiful fate of a physical theory is to point the way to the establishment of a more inclusive theory, in which it lives on as a limiting case." (18, p. 52.) In fact, in the advance from the special to the general theory of relativity, we have only a verification of the same principle of the construction of concepts of natural science that is found in the advance from classical mechanics to the special theory of relativity. The constants of measurement and of the theory of nature in general are shifted and magnitudes, which were regarded as absolute from the earlier standpoint, are again, with the gaining of a new theoretical unit of measurement, made into merely relative determinations valid only under definite conditions. While classical mechanics, like the special theory of relativity, distinguished between certain reference bodies relatively to which the laws of nature were valid and certain relatively to which they were not, this distinction is now cancelled. The expression of the universal physical laws is freed from any connection with a particular system of coördinates or with a certain group of such systems. To be expressed the laws of nature always require some definite system of reference; but their meaning and value is independent of the individuality of this system and remains self-identical whatever change the latter may undergo.

Only with this result do we reach the real center of the general theory of relativity. Now we know where lie its truly ultimate constants, its cardinal points, around which it causes phenomena to revolve. These constants are not to be sought in particular given things, which are selected as chosen systems of reference from all others, such systems as the sun was to Copernicus and as the fixed stars were for Galilei and Newton. No sort of things are truly invariant, but always only certain fundamental relations and functional dependencies retained in the symbolic language of our mathematics and physics, in certain equations. This result of the general theory of relativity, however, is so little a paradox from the standpoint of the criticism of knowledge, that it can rather be regarded as the natural logical conclusion of an intellectual tendency characteristic of all the philosophical and scientific thought of the modern age.[8] To the popular view and its habits of thought the radical resolution of "things" into mere relations remains as ever suspicious and alienating, for this view believes that it would lose with the thing-concept the one sure foundation of all objectivity, of all scientific truth. And thus, from this side not so much the positive as the negative aspect of the theory of relativity has been emphasized; what it destroys, not what it constructs has been comprehended. But it is remarkable to find this interpretation not only in the popular expositions of the theory of relativity but in the investigations of its general "philosophical" significance; and to meet in the latter also the view that it brings an element of subjective arbitrariness into the formulation of the laws of nature and that, along with the unity of space and time, the unity of the concept of nature is destroyed. In truth, as closer consideration shows, the theory of relativity is characterized through-

[8] Here, indeed, I can only make this assertion in a general way; for its proof I must refer to the more specific explanation in my work *Substanzbegriff und Funktionsbegriff*. (8, pp. 148-310).

out by the opposite tendency. It teaches that to attain an objective and exact expression of natural process, we cannot take without further consideration the space and time values, gained by measurement within a definite system of reference as the only and universal values, but that we must, in scientifically judging these measurements, take account of the state of motion of the system from which the measurement is made. Only when this is done can we compare measurements which have been made from different systems. Only those relations and particular magnitudes can be called truly objective which endure this critical testing, that is, which maintain themselves not only for one system but for all systems. That not only are there such relations and values, but that there must be such, in so far as a science of nature is to be possible, is precisely the doctrine the theory of relativity sets up as a postulate. If we start, as practically we must do at first, from a definite system of measurement, we must bear in mind that the empirical values, which we gain here, do not signify the final natural values but that, to become such, they must undergo an intellectual correction. What we call the system of nature only arises when we combine the measurements, which are first made from the standpoint of a particular reference body, with those made from other reference bodies, and in principle with those made from all "possible" reference bodies, and bring them ideally into a single result. How there can be found in this assertion any limitation of the "objectivity" of physical knowledge is not evident; obviously it is meant to be nothing but a definition of this very objectivity. "But it is clear," says Kant, "that we have only to do with the manifold of our presentations and that X, which corresponds to them (the object), since it is to be something distinct from all our presentations, is for us nothing; the unity, which makes the object necessary, can be nothing else than the formal

unity of consciousness in the synthesis of the manifold of presentations. Thus we say: we know the object when in the manifold of intuition we have produced synthetic unity." The object is thus not gained and known by our going from empirical determinations to what is no longer empirical, to the absolute and transcendent, but by our unifying the totality of observations and measurements given in experience into a single complete whole. The theory of relativity shows the whole complexity of this task; but it retains the postulate of the possibility of such a system all the more strenuously and points out a new way to satisfy it. Classical mechanics believed itself at the goal too soon. It clung to certain reference bodies and believed that it possessed, in connection with them, measures in some way definitive and universal, and thus absolutely "objective." For the new theory, on the contrary, true objectivity never lies in empirical determinations, but only in the manner and way, in the function, of determination itself. The space and time measurements in each particular system are relative; but the truth and universality, which can be gained nevertheless by physical knowledge, consist in the fact that all these measurements correspond mutually and are coördinated with each other according to definite rules. More than this indeed knowledge cannot achieve, but it cannot ask for more, if it understands itself. To wish to know the laws of natural processes independently of all relation to any system of reference, is an impossible and self-contradictory desire; all that can be demanded is that the content of these laws not be dependent on the individuality of the system of reference. It is precisely this independence of the accidental standpoint of the observer that we mean when we speak of the "natural" object and the "laws of nature" as determinate in themselves. Measurements in *one* system, or even in an unlimited number of "justified" systems would

in the end give only particularities, but not the true "synthetic unity" of the object. The theory of relativity teaches, first in the equations of the Lorentz-transformation and then in the more far reaching substitution formulae of the general theory, how we may go from each of these particularities to a definite whole, to a totality of invariant determinations. The anthropomorphism of the natural sensuous picture of the world, the overcoming of which is the real task of physical knowledge,[9] is here again forced a step further back. The mechanical view of the world thought to have conquered it, when it resolved all being and natural process into motion and thus put every where pure magnitudes in place of qualitative elements of sensation. But now it is seen that precisely the determination of these values, the measurements, which it applies to motions, are still bound to certain limiting presuppositions. Reflection on the manner in which we make empirical measurements of space and time shows how anthropomorphism reaches into this field that was thought withdrawn from it in principle. It is, as it were, this earthly remainder still belonging to classical mechanics with its assumption of finite fixed reference bodies and motionless inertial systems, from which the theory of relativity seeks to free itself. The conceived unit of connection determined by a system of mathematical equations here takes the place of any sensuously given, and also sensuously conditioned, unit of measurement. As is seen, there is involved here not a cancellation but a critical correction of the empirical concept of objectivity, by which a correction of our empirical spatial and temporal measures and their transformation into the *one* system of natural laws are gained.

We are brought to the same outcome by consideration of the historical problems out of which the theory of rela-

[9] *Cf.* Planck (66) p. 6ff. and (67) p. 74.

tivity has grown. To give the propositions of abstract mechanics, especially the principle of inertia a definite physical meaning had been attempted repeatedly by trying to point out some empirical systems for which they would possess strict validity. But these attempts were all thwarted, in particular, by the discovery of the motion of the solar system and of the fixed stars; to find a fixed and clear empirical meaning for the equations of the Galilei-Newtonian mechanics, nothing remained save to postulate, with Carl Neumann, an absolutely motionless body α at some unknown place in space. But such a postulate of the existence of a particular physical object, a body which can never be discovered by observation, remains the strangest anomaly, from the epistemological standpoint. (8, p. 238ff.) The absolutely motionless ether too, which seemed for a time to offer the lacking physical reference system of the Galilei-Newtonian mechanics, showed itself unsuited to this purpose; since the negative outcome of Michelson's investigation the question seemed to be decided here also. At this point, as has been seen, the theory of relativity begins. It makes a virtue out of the difficulty into which philosophical thought had fallen in its attempt to find a particular privileged system of coördinates. Experience had shown that there is no such system, and the theory, in its most general interpretation, makes it a postulate that there *cannot* and *must* not be such. That, for the physical description of the processes of nature, no particular reference body is to be privileged above any other is now made a principle. "In classical mechanics, as well as the special theory of relativity," says Einstein," a distinction is drawn between reference bodies K relatively to which the laws of nature are valid and reference bodies K′ relatively to which they are not valid. With this state of affairs no consistently thinking man can be satisfied. He asks: how is it possible that certain reference bodies (and their states

of motion) are privileged over other reference bodies (and their states of motion)? In vain, I seek in classical mechanics for something real to which I might trace the difference in the behavior of the body with reference to the systems of reference K and K'." (18, p. 49.) In this argument from the principle of *insufficient reason,* the physicist seems to move on slippery ground. One is inevitably reminded of the argument of Euler, who thought that he *proved* the principle of inertia of classical mechanics by explaining that, if a body changed its state of motion without the influence of external forces, there would be no reason why it should choose any particular change of magnitude and direction of its velocity. (23.) The circle involved here, namely, that "the state of motion" of a body is assumed to be a determinate magnitude, while it is only defined as such by the law of inertia itself, is easily seen. In Einstein's appeal to the "principle of reason," there is doubtless involved a more general and deeper epistemological motive. If we assume that the final objective determinations, which our physical knowledge can reach, *i. e.,* the laws of nature, are provable and valid only for certain chosen systems of reference, but not for others, then, since experience offers no certain criterion that we have before us such a privileged reference system, we can never reach a truly universal and determinate description of natural processes. This is only possible if some determinations can be pointed out, which are indifferent to every change in the system of reference taken as a basis. Only these relations can we call laws of nature, *i. e.,* ascribe to them objective universality, whose form is independent of the particularity of our empirical measurements of the special choice of the four variables x_1, x_2, x_3, x_4, which express the space and time parameters. In this sense, one could conceive the principle of the universal theory of relativity, that the universal laws of nature are

not changed in form by arbitrary changes of the space-time variables, as an analytic assertion; as an explanation of what is meant by a "universal" law of nature. But the demand, that there must in general be such ultimate invariants, is synthetic.

In fact, it can be shown that the general doctrine of the invariability and determinateness of certain values, which is given first place by the theory of relativity, must recur in some form in any theory of nature, because it belongs to the logical and epistemological nature of such a theory. To start from the picture of the world of general energetics—Leibniz, in establishing the law of the "conservation of *vis viva*" as a universal law of nature, referred to this logical element in it. He first defines the *vis viva* of a physical system as a quantity of work; he determines that forces are to be called equal, when they are able to perform equal mechanical work, no matter what their properties may be in detail; thus if they produce an equal degree of tension in an equal number of elastic springs, raise an equal weight to the same height, communicate to an equal number of bodies the same amount of velocity, *etc.* In this definition it is assumed that measurement of the *vis viva* by different systems of measurement will give results equivalent to each other, and thus that forces, which, when measured by a certain effect, prove to be equal or in a definite relation of greater or smaller, will retain this same relation if we measure them by any other effect. If this were not the case, Leibniz adds, and did there result a different relation of forces according to the different effect which one uses as a measure, nature would be without laws; the whole science of dynamics would be superfluous; and it would not be possible to measure forces, for forces would have become something indeterminate and contradictory, *quiddam vagum et absonum.* (42, III, 208ff.; VI, 209f.; *cf.* 5, p.

305ff.) The same process of thought has been repeated on broader physical lines in the discovery and grounding of the modern principle of energy. Here, too, the energy of a material system in a certain state was defined—*e. g.*, by W. Thompson—first as the amount of all the effects, expressed in mechanical units of work, called forth outside the system when the system passes in any way from its state into a definite but arbitrarily defined state of nullity. This explanation at first leaves it entirely undecided as to whether there exists a determinate value of what is here called "energy," *i. e.*, whether the results of the measurement of the amount of work of a system turn out the same or differently according to the method of bringing the system from the given state into a definite state of nullity. But that this determinateness in fact exists, that there always results the same amount of energy no matter what effect we use as the measure of work and what type of transition we choose, is precisely what the principle of the conservation of energy affirms. This affirms nothing else and has no other physically comprehensible meaning than that the amount of all the effects, measured in units of mechanical work, which a material system calls forth in its external environment, when it passes from a definite state in any arbitrary manner to an arbitrarily defined state of nullity, has a determinate value, and is thus independent of the type of transition. If this independence did not exist—and that it exists only experience can teach us—it would follow that what we called "energy" is not an exact physical determination; energy would not be a universal constant of measurement. We would have to seek for other empirical values to satisfy the fundamental postulate of determinateness. But it holds, conversely, that if energy is once established as a constant of measurement, it thus becomes a constant of nature also, a "concept of a definite object." Now from a physical

standpoint a "substantial" conception of energy can be carried through without arousing suspicion; energy can be regarded as a sort of "reserve supply" of the physical system, the quantity of which is completely[10] determined by the totality of the magnitudes of the states, which belong to the system involved. From the epistemological standpoint, it must be remembered that such an interpretation is nothing more than a convenient expression of the relations of measurement, that alone are known, an expression which adds to them nothing essential. The unity and determinateness of measurement can be immediately understood and expressed as the unity and determinateness of the object, precisely because the empirical object means nothing but a totality of relations according to law. It follows from this analogy from a new angle that the advance in "relativization" which takes place in the theory of relativity, represents no contrast to the general task of objectification, but rather signifies one step in it, since, by the nature of physical thought, all its knowledge of objects *can* consist in nothing save knowledge of objective relations. "Whatever we may know of matter," here, too, we can cite the *Critique of Pure Reason,* "is nothing but relations, some of which are independent and permanent and by which a certain object is given us." (34, p. 341; *cf.* Müller's Trans. p. 232.) The general theory of relativity has shifted these "independent and permanent relations" to another place by breaking up both the concept of matter of classical mechanics and the concept of the ether of electrodynamics; but it has not contested them as such, but has rather most explicitly affirmed them in its own invariants, which are independent of every change in the system of reference. The criticism made by the theory of relativity of the physical concepts of objects springs thus from the same method of scientific thought,

[10] In more detail in Planck (63) p. 92ff.

which led to the establishment of these concepts, and only carries this method a step further by freeing it still more from the presuppositions of the naïvely sensuous and "substantialistic" view of the world. To grasp this state of affairs in its full import we must go back to the general epistemological questions offered to us by the theory of relativity; we must go back to the transformation of the *physical concept of truth* involved in it by which it comes into direct contact with the fundamental problem of logic.

ERNST CASSIRER.*

UNIVERSITY OF HAMBURG.

* This article is a selection from a translation of Professor Ernst Cassirer's *Zur Einstein'schen Relativitätstheorie.* A translation of Professor Cassirer's *Substanzbegriff und Funktionsbegriff,* with the whole of the former essay as a supplement, will be published in the course of the year by THE OPEN COURT PUBLISHING COMPANY as one volume under the title *Substance and Function.* The translators are W. Curtis Swabey, Ph.D., and Marie Collins Swabey, Ph.D.